Models of magnetic anisotropy for non oriented steel sheets dedicated to finite element method

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Even non oriented steel sheets present anisotropic behavior. Our investigation consists of developing two models based on rotational measurements to consider these magnetic properties. The direct method models both components of the magnetic field with the ones of the magnetic flux density. The indirect method determines the energy density as a function of the magnetic flux density. The magnetic field is then calculated by differentiating the energy density with respect to the magnetic flux density. Both models are finally validated by comparing measured and computed values of the magnetic field.

***Index Terms*—Magnetic properties, Non-Oriented steel sheets, Anisotropy**

I. INTRODUCTION

THE mangetization properties of soft magnetic materials differ with the excitation field direction. Although this anisotropy can be deliberately performed with Grain Oriented steel sheets, the magnetization curves in rolling (RD) and transverse (TD) directions differ significantly for Non Oriented (NO) steel sheets [1], [2]. Hence reliable models of this phenomenon are required in numerical analyzes such as finite element methods.

Three main intrinsic phenomena entail anisotropic characterization of body centered cubic iron [3]: the *shape anisotropy* depends on the shape of ferromagnetic crystal and its demagnetization field, the *magnetocrystalline* anisotropy alters the magnetization properties relying on the spin-orbit interactions of the crystal atomic structure, and the *magnetostriction* enhances an easy direction of the magnetization by deforming the domains.

Since the magnetic anisotropy infers a dependance of reluctivity on both amplitude and direction of the applied flux density, its models can be developed by interpolating the reluctivity between two adjacent $\mathbf{B}-\mathbf{H}$ curves extracted from measurements. The magnetic field can be decomposed into a purely isotropic component and an effective anisotropic one. This model can be implemented into a Garlekin's formulation [4].

Based on energy/coenergy density principle [5], Péra et al. [6] expand a phenomenological model on GO sheets which needs only magnetization curves in RD and TD directions. Although, their computational implementation requires some differentiations based on interpolation, their model fits well with alternating flux measurements for various directions in the range of 200 A/m to 30 kA/m. However, the four magnetization modes described in [7] are not fully described by this phenomenological approach, so measurements in more directions are needed to characterize steel sheets completely.

In this paper, we focus on models of magnetic properties dedicated for 2D finite element methods with the magnetic vector potential $\mathbf{A}$. Its functional calculus is given by [8]:

$$\mathcal{F} = \int_{\Omega} \left[\int_0^{\mathbf{B}} \mathbf{H}^{\mathrm{T}}\, d\mathbf{B} - \mathbf{S}\,\mathbf{A} \right] dV \tag{1}$$

where $\mathbf{B}$ and $\mathbf{H}$ are the magnetic flux density and magnetic field strength respectively, $\mathbf{S}$ is the current density source and Ω is the considered volume. Coupled with a Newton Raphson method, the estimated magnetic vector potential is updated after each iteration k by solving the following equation :

$$\mathbf{J}_k \left[\mathbf{A}_{k+1} - \mathbf{A}_k\right] = \mathbf{R}_k \tag{2}$$

where the residual vector $\mathbf{R}$ and Jacobian matrix $\mathbf{J}$ are given by:

$$\begin{aligned} \mathbf{R}_i &= -\int_{\Omega} \nabla \times (N_i\, e_z)^{\mathrm{T}}\, \mathbf{H}(\mathbf{B}) - \mathbf{J}\, N_i\, dV \\ \mathbf{J}_{i\,j} &= -\int_{\Omega} \nabla \times (N_i\, e_z)^{\mathrm{T}} \left[\frac{\partial \mathbf{H}}{\partial \mathbf{B}}\right] \nabla \times (N_j\, e_z)\, dV \end{aligned} \tag{3}$$

with N the shape function of the finite element method and e_z the unit vector in the z direction.
The convergence of the Newton Raphson method lays on the matrix $\partial\mathbf{H}/\partial\mathbf{B}$ which should be positive definite. So the model of magnetic property should respect this property and it should represent with enough accuracy the measured magnetic magnitudes. Hence, we propose to model magnetic anisotropy of NO sheets with two methods:

- Direct method: the components of $\mathbf{H}$ are modeled as a function of the components of the magnetic flux density;
- Indirect approach: from integration of $\mathbf{H}(\mathbf{B})$ for every direction of $\mathbf{B}$, the energy density F is represented with the components of $\mathbf{B}$. The magnetic field is deduced by differentiating the energy density with respect to the components of $\mathbf{B}$.

II. MODELS OF MAGNETIC ANISOTROPY

Measurements have been performed in a single NO steel sheet under rotational flux density at 10 Hz. Every component of $\mathbf{B}-\mathbf{H}$ loci are measured with 1 000 points with 18 different amplitudes of magnetic flux density from 0.1 T to 1.75 T. In order to extract the anhysteretic curves for every direction, the anisotropic effect is analyzed after removing the losses for each locus. The losses can be removed by canceling the phase shift between both fundamental components of magnetic flux density and magnetic field.

A. Direct model of the components of H

In order to ensure that the matrix $\partial\mathbf{H}/\partial\mathbf{B}$ is positive definite, the amplitude H of the magnetic field should be strictly monotonous with the amplitude B of the flux density. Thus the amplitude H is fitted with the following analytical function:

$$H(B,\,\phi_B) = C(\phi_B)\,\left[\exp\left(\tau(\phi_B)\,B\right)-1\right] + D(\phi_B)\,\sqrt[4]{B} \quad (4)$$

where the parameters C, τ and D are fitted by a cubic spline interpolation for every direction ϕ_B of the magnetic flux density.

The direction ϕ_H of the magnetic field, which is also dependent of both components of $\mathbf{B}$, is fitted with a bi-cubic surface spline interpolation.

B. Indirect model based on the magnetic energy density F

For every direction of the flux density, the magnetic energy density F is calculated by integrating the anhysteretic curve $H(B)$. This integration is performed by first interpolating the anhysteretic curves with a cubic spline interpolation and then integrating these splines. After calculating the energy density at its corresponding measurements of B, the magnetic energy density is fitted with the following analytical function:

$$F(B,\,\phi_B) = \alpha(\phi_B)\,\left[\exp\left(\beta(\phi_B)\,B\right)-1\right] + \gamma(\phi_B)\,B \quad (5)$$

where the parameters α, β and γ are fitted by a cubic spline interpolation for every direction ϕ_B of the magnetic flux density.

The components of the magnetic field strength H and ϕ_H are determined by differentiating the energy density with respect to the components of the magnetic flux density B and ϕ_B respectively. In $x-y$ coordinate system, the components of $\mathbf{H}$ are determined by:

$$\begin{aligned} h_x &= \frac{b_x}{\sqrt{{b_x}^2+{b_y}^2}}\frac{\partial F}{\partial B} - \frac{b_y}{{b_x}^2+{b_y}^2}\frac{\partial F}{\partial \phi_B} \\ h_y &= \frac{b_y}{\sqrt{{b_x}^2+{b_y}^2}}\frac{\partial F}{\partial B} + \frac{b_x}{{b_x}^2+{b_y}^2}\frac{\partial F}{\partial \phi_B} \end{aligned} \quad (6)$$

III. RESULTS AND DISCUSSION

In order to validate the proposed models, the $\mathbf{H}$ loci are estimated by both models and compared with the measurements. Due to its small impact, $\partial F/\partial\phi_B$ has been neglected in the indirect model. Figure 1 shows that the proposed models can reproduce the measured locus for different amplitudes of the magnetic flux density. The direct method provides the best accuracy especially with high amplitude of the magnetic flux density. However, the indirect method presents some smoother $\mathbf{H}$ loci. So the indirect method could be employed to ease the convergence of the direct method for its implementation in finite element method.

In the final paper, both models will be improved. Moreover, their implementation in 2D finite element method will be discussed.

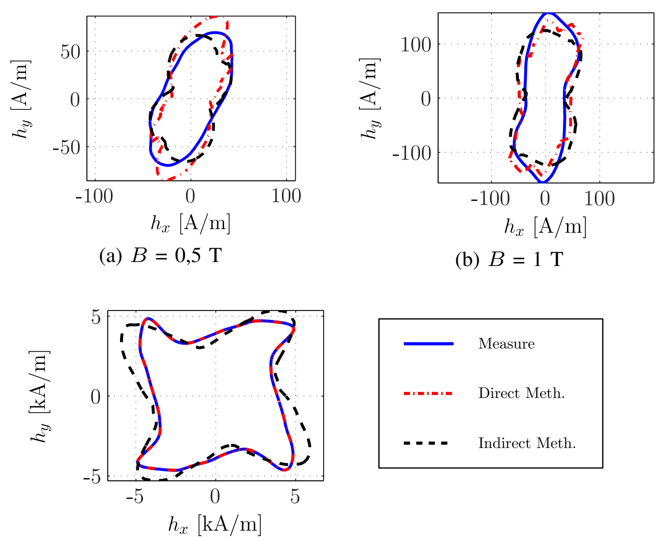

Fig. 1. Measurements and models of $\mathbf{H}$ loci

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